in which x represents a side of the given cube. The labor required to perform the indicated integrations is *enormous*—enough to discourage the most enthusiastic mathematical genius.

NOTE—Since the parenthetical expressions in $\triangle P_1P_2O=\frac{1}{2}\sqrt{[(vx-uy)^2+(wx-uz)^2+(wy-vz)^2]}$ represent respectively 2(Area of the projections of of $\triangle OP_2P_1$) on the co ordinate planes XY, ZX, the result of problem 2, Average and Probability in April No. of the MONTHLY, substituted in these expressions gives $\triangle P_1P_2O=\frac{11}{108}a^2\sqrt{3}$ as the required average area.—MATZ.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey. Va All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

11. Proposed by CHARLES E. MYERS, Canton Ohio

"Assuming the earth's orbit to be a circle, if a comet move in a parabola around the sun and in the plane of the earth's orbit, show that the comet cannot remain within the earth's orbit longer than 78 days."

I. Solution by WILLIAM HOOVER, A. M Ph. D., Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio

Let $4a$=the latus rectum of the comet's orbit, ρ=its distance at any time from the sun, p=the perpendicular from the sun upon a tangent to the comet's path, F=the attractive force of the sun, h=the double area described by ρ in a unit of time, and θ the angular co-ordinate corresponding to ρ. Then $F=\dfrac{h^2}{p^3}\dfrac{dp}{d\rho}$ ....(1), and from the parabola $\dfrac{2}{p^3}\dfrac{dp}{d\rho}=\dfrac{1}{a\rho^2}$; $\therefore$ $F=\dfrac{h^2}{2a\rho^2}$....(2).

Let $F=\phi$, when $\rho=1$; then $h=\sqrt{(2a\phi)}$....(3), and $F=\dfrac{\phi}{\rho^2}$....(4).

If r=the radius of the earth's orbit, and v=the velocity of the earth, $v^2=rF=\dfrac{\phi}{r}$, or $v=\sqrt{\dfrac{\phi}{r}}$. Then $\theta r\div v=\dfrac{\theta(r)^{\frac{3}{2}}}{\sqrt{\phi}}$ =the time required for the earth to describe the arc subtending the angle θ at the sun....(e).

For the comet, $dt=\dfrac{\rho^2 d\theta}{h}$ ....(5), and from the parabola,

$\rho=\dfrac{2a}{1+\cos\theta}=\dfrac{a}{\cos^2\frac{1}{2}\theta}$....(6). This gives $\rho^2 d\theta=\dfrac{a^2 d\theta}{\cos^4\frac{1}{2}\theta}$, and then (5) gives

$$t=\frac{2a^2}{h}\int\frac{d\theta}{\cos^4\frac{1}{2}\theta}=\frac{2a^2}{\sqrt{2a\phi}}\left\{\tan\tfrac{1}{2}\theta+\tfrac{1}{3}\tan^3\tfrac{1}{2}\theta\right\}....(7).$$

The circle $\rho=r$ intersects (6) in a point given by $\cos\frac{1}{2}\theta=\sqrt{\frac{a}{r}}$ or

$\tan\frac{1}{2}\theta=\sqrt{\frac{r-a}{a}}$; we then have $2t=2a^{\frac{3}{2}}\sqrt{\frac{2}{\phi}}\left\{\sqrt{\frac{r-a}{a}}+\frac{1}{3}\sqrt{\frac{(r-a)^3}{a^3}}\right\}$

$=\frac{2}{3}\sqrt{\frac{2}{\phi}}(r+2a)\sqrt{(r-a)}\ldots.(8)$. This must be a maximum.

Equating $\frac{dt}{dr}$ to zero and solving for r, we find $r=2a$.

$\therefore\ \theta=\frac{\pi}{2}$, and (e) becomes $\frac{2\pi a\sqrt{2a}}{\sqrt{\phi}}$ and (8), $\frac{4}{3}a\sqrt{\frac{2a}{\phi}}$. These give for the greatest part of the earth's year during which a parabolic comet can remain in the earth's orbit is $\frac{2a}{3\pi}$, or about 78 days.

II. **Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virgina.**

Let $r=a$ be the equation to the earth's orbit.

$r=\frac{2d}{1+\cos\theta}$ be the equation to the comet's orbit.

Also $u=\frac{1}{r}$, for the circle $\frac{du}{d\theta}=\frac{d^2u}{d\theta^2}=0$, for the parabola $\frac{du}{d\theta}$

$=-\frac{\sin\theta}{2d}$, $\frac{d^2u}{d\theta^2}=-\frac{\cos\theta}{2d}$, but P=force of attraction$=h^2u^2\left\{\frac{d^2u}{d\theta^2}+u\right\}$

$\therefore\ P=\frac{h^2u^2}{a}=\frac{h^2}{a^3}$ for circle. $P=h^2u^2\left\{\frac{1+\cos\theta}{2d}-\frac{\cos\theta}{2d}=\frac{h^2}{2dr^2}\right.$

for the parabola at the point of intersection of the two curves $r=a$ and the values of P for each are equal.

$\therefore\ \frac{h^2}{a^3}=\frac{h^2}{2dr^2}=\frac{h^2}{2da^2}$, $\therefore\ a=2d$. Also at the intersection

$r=\frac{2d}{1+\cos\theta}=a$, but $a=2d$. $\therefore\ \cos\theta=0$, and $\theta=\frac{\pi}{2}$.

The tirms of describing any given angle is obtained from the formula $r^2\frac{d\theta}{dt}=h$. $\therefore\ dt=\frac{r^2}{h}d\theta$, we found θ above to be $\frac{\pi}{2}$ measured from the vertex of the parabola. Hence the time the comet is within the earth's orbit is

$t=\frac{2a^2}{h}\int_0^{\frac{\pi}{2}}\frac{d\theta}{(1+\cos\theta)^2}=\frac{4a^2}{3h}=\frac{4}{3}\sqrt{\frac{a^3}{\mu}}$ where μ=absolute force. The periodic time for the earth is $t=4\int_0^{\frac{\pi}{2}}\frac{r^2}{h}d\theta=\frac{4a^2}{h}\int_0^{\frac{\pi}{2}}d\theta=\frac{2\pi a^2}{h}=2\pi\sqrt{\frac{a^3}{\mu}}=$ year.

Let x=time the comet is within the earth's orbit;

then $2\pi\sqrt{\frac{a^3}{\mu}}:\frac{4}{3}\sqrt{\frac{a^3}{\mu}}=1$ year : x. $\therefore$ $x=\left(\frac{2}{3\pi}\right)^{th}$ part of a year

$=\frac{2}{3\pi}\times365\frac{1}{4}$ days$=77.208+$days.

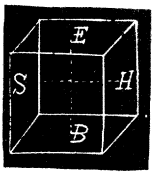
PROBLEMS.

20. Proposed by SAMUEL HART WRIGHT, M. D., M. A., Ph. D., Penn Yan, Yates County, New York.

When does The Dog-Star and the Sun rise together in Latitude 42° 30′ North=λ? Given the R. A. of Sirius=6 h. 40 m. 30 sec. and its Dec.= 16° 33′ 56″ S.

21. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

Solve (if possible) the following: A cube whose edge is a feet revolves on both axes—EB and SH—at the same number of revolutions per minute.



What is the volume of the figure generated, (a) when the center of the cube remains in one place, (b) when the center of the cube moves b feet in a straight line in a minute?

QUERIES AND INFORMATION

Conducted by J. M. COLAW, Monterey. Va. All contributions to this department should be sent to him.

Notes on Counsellor Dolman's Remarks In April Number.
By Professor John N. Lyle.

[Received May, 1894.]

Says Counsellor Dolman—"According to Lobatschewsky the angle-sum of a rectilineal triangle decreases as the area of the triangle increases, but is always less than two right angles."

What is a *rectilineal* triangle? The answer is, one whose sides are straight lines. "A triangle can be formed of three straight lines joining any three points." As three points determine the position of a plane, the surface of a rectilineal triangle is a plane.